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NAHE-Based String Models With $SU(4) \times SU(2) \times U(1)$ $SO(10)$ Subgroup

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Abstract

The orbifold GUT doublet-triplet splitting mechanism was discussed in 1994 in the framework of the NAHE-based free fermionic models in which the $SO(10)$ GUT symmetry is broken to $SO(6) \times SO(4)$, $SU(3) \times SU(2) \times U(1)^2$, or $SU(3) \times U(1) \times SU(2)^2$. In this paper we study NAHE-based free fermionic models in which the $SO(10)$ symmetry is broken at the string level to $SU(4) \times SU(2) \times U(1)$. In addition to the doublet-triplet splitting this case also has the advantage of inducing the doublet-doublet splitting already at the string level. We demonstrate, however, that NAHE-based models with $SU(4) \times SU(2) \times U(1)$ $SO(10)$ subgroup are not viable. We show that, similarly to the LRS models, and in contrast to the FSU5, PS and SLM models, the SU421 case gives rise to models without an anomalous $U(1)$ symmetry, and discuss the different cases in terms of their $N = 4$ origins.

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1 Introduction

Grand unified extensions of the Standard Particle Model are well supported by the phenomenological Standard Model data. This is reflected by the particle charges, by the extrapolation of the gauge, and heavy generation mass, parameters, and by the suppression of proton decay and neutrino masses. The most appealing GUT embedding is achieved with an $SO(10)$ gauge group in which each of Standard Model generations is embedded in a single 16 $SO(10)$ representation. However, it is clear that a more complete understanding of the Standard Model parameters necessitates the incorporation of gravity in the unification program. This conclusion follows from the fact that flavor and mass do not originate from Grand Unified Theories. These origins must then be sought in the scale above the GUT scale, namely the Planck scale, and therefore in a theory of quantum gravity.

String theory is the unique contemporary framework that enables detailed studies of the gauge–gravitational unification. Given the success of the Standard Model and Grand Unified Theories it is natural to seek string models that preserve the GUT embedding of the Standard Model spectrum. The GUT embedding may be realized at the string level and is not necessarily present in the effective low energy field theory. Most compelling in this context are models that preserve the $SO(10)$ embedding of the Standard Model spectrum.

Following the string duality developments we now believe that the string theories are perturbative limits of a still unknown more fundamental theory. In this respect, we should think of the string theories as probes of the properties of the more basic structures. In this view, different string limits may be advantageous in probing different characteristics of the underlying theory. So, for example, the issue of supersymmetry breaking and dilaton stabilization may be better addressed in the type I limit. On the other hand the limit that enables the $SO(10)$ embedding of the Standard Model spectrum is the heterotic–string limit as it is the only limit that produces the chiral 16 representation of $SO(10)$ in the perturbative massless spectrum. In addition to the desirable GUT embedding of the Standard Model spectrum one must also impose that the perturbative massless spectrum contains three chiral generations.

A class of three generation heterotic–string compactifications that preserves the $SO(10)$ embedding of the Standard Model spectrum was constructed in the so–called free fermionic formulation. The existence of models in this class which produce solely the spectrum of the Minimal Supersymmetric Standard Model in the observable massless sector was further demonstrated [1]. The free fermionic models correspond to $Z_2 \times Z_2$ orbifold compactification at a maximally symmetric point in the Narain moduli space and additional Wilson lines. The realistic features of the free fermionic models are rooted in the underlying $Z_2 \times Z_2$ orbifold structure, and it is of course of much interest to examine whether there exists a dynamical reason why this class of compactifications would be selected.

The $SO(10)$ symmetry is broken to one of its subgroups by the free fermion boundary condition basis vectors, that are equivalent to Wilson lines in the orbifold formulation. To date $SO(10)$ subgroups that have been studied include the flipped $SU(5)$ (FSU5) [2]; the standard-like models (SLM) [3, 4, 1]; the Pati–Salam models (PS) [5]; and the left–right symmetric models (LRS) [6]. Many of the phenomenological properties of the models naturally relate to the $SO(10)$ subgroup which remains unbroken. For example, the stringy doublet–triplet mechanism of ref [7] operates in the SLM, PS, and LRS models but not in the FSU5 models*. Similarly, the top–bottom mass splitting mechanism of ref. [10] operates in the SLM and FSU5 models, but not in the PS or LRS models. Another important example is the relation between the presence of an anomalous $U(1)$ and the choice of the unbroken $SO(10)$ subgroup. In the case of the FSU5, SLM and PS one always finds an anomalous $U(1)$, whereas LRS models sometimes yield three generation models that are free from any Abelian and non–Abelian anomalies. This aspect is particularly intriguing as it relates to the issue of supersymmetry breaking in the string models. Specifically, it was demonstrated in ref. [11] that in LRS models that do contain an anomalous $U(1)$ there exist only non–Abelian flat directions, whereas Abelian flat–directions do not exist.

To date, the class of free fermionic models with unbroken $SU(4) \times SU(2) \times U(1)$ (SU421) $SO(10)$ subgroup has not been analyzed in the literature. In this paper we undertake this analysis, and by that complete the analysis of all the possible unbroken $SO(10)$ subgroups. We demonstrate that in fact, with the NAHE set basis vectors [12], this choice cannot produce realistic spectrum. Specifically, we demonstrate that this choice of $SO(10)$ symmetry breaking pattern necessarily results in incomplete $SO(10)$ multiplets and hence does not produce the Standard Model chiral matter. Additionally, we show that, similarly to the LRS models, SU421 models can produce anomaly free, or anomalous, $U(1)$ models, depending on the initial $N = 4$ vacuum. The existence or absence of an anomalous $U(1)$ hinges on the important issue of supersymmetry breaking. In ref. [11] we argued that the absence of Abelian flat directions in anomalous LRS models indicates that supersymmetry is broken hierarchically in the models by hidden sector gaugino and matter condensation. However, here we find that, unlike the LRS models, singlet flat directions also exist in the SU421 models that do contain an anomalous $U(1)$.

2 The supersymmetric $SU(4) \times SU(2) \times U(1)$ model

In this section we briefly summarize the field theory structure of the type of models that we aim to construct from string theory in this paper. The observable

*Similar doublet–triplet splitting mechanisms were also discussed in the context of other Calabi–Yau and orbifold constructions[8]. The fermionic models afforded the understanding of the splitting in terms of the GSO projections and the world–sheet fermionic boundary conditions, that correspond to action on the internal dimensions in the corresponding bosonic language [7]. Alternatively, an attractive field theory mechanism is obtained in the flipped $SU(5)$ [9].

sector gauge symmetry we seek is $SU(4)_C \times SU(2)_L \times U(1)_R$. Such models are reminiscent of the PS type string models, but differ from them by the fact that the $SU(2)_R$ gauge group is broken to $U(1)_R$ already at the string level. Similar to the PS models, the $SU(4)_C \times SU(2)_L \times U(1)_R$ models possess the $SO(10)$ embedding. The quarks and leptons are accommodated in the following representations:

$$F_L^i = (4, 2, 0) = (3, 2, \frac{1}{3}, 0) + (1, 2, -1, 0) = \begin{pmatrix} u \\ d \end{pmatrix}^i + \begin{pmatrix} \nu \\ e \end{pmatrix}^i, \quad (2.1)$$

$$U_R^i = (\bar{4}, 1, -\frac{1}{2}) = (\bar{3}, 1, -\frac{1}{3}, -\frac{1}{2}) + (1, 1, +1, -\frac{1}{2}) = u^{ci} + N^{ci}, \quad (2.2)$$

$$D_R^i = (\bar{4}, 1, +\frac{1}{2}) = (\bar{3}, 1, -\frac{1}{3}, +\frac{1}{2}) + (1, 2, +1, +\frac{1}{2}) = d^{ci} + e^{ci}, \quad (2.3)$$

where the first and second equalities display the decomposition under $SU(4)_C \times SU(2)_L \times U(1)_R$ and under $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_R$, respectively, and the weak-hypercharge is given by

$$U(1)_Y = \frac{1}{2}U(1)_{B-L} + U(1)_R. \quad (2.4)$$

Hence, F_L produces the quark and lepton electroweak doublets whereas, U_R and D_R produces the right-handed electroweak singlets. The two low energy supersymmetric Higgs superfields associated with the Minimal Supersymmetric Standard Model, h^d and h^u , are given by,

$$h^d = (1, 2, -\frac{1}{2}), \quad (2.5)$$

$$h^u = (1, 2, +\frac{1}{2}). \quad (2.6)$$

The heavy Higgs multiplets that break $SU(4)_C \times U(1)_R$ to the Standard Model group factors $SU(3) \times U(1)_Y$ are given by the fields

$$H = (\bar{4}, 1, +1) \quad (2.7)$$

$$\bar{H} = (4, 1, -1) \quad (2.8)$$

The SU421 string models can also contain Higgs fields that transform as

$$(6, 1, 0) = (3, 1, \frac{1}{3}, 0) + (\bar{3}, 1, -\frac{1}{3}, 0),$$

that originates from the vectorial **10** representation of $SO(10)$. These color triplets mediate proton decay through dimension five operators, and consequently must be sufficiently heavy to insure agreement with the proton lifetime. An important advantage of the SU421 symmetry breaking pattern, with $SO(10) \rightarrow SO(6) \times SO(4) \rightarrow SU(4) \times SU(2) \times U(1)$ at the string construction level, is that these color triplets

may be projected out by the GSO projections [7], and therefore need not be present in the low energy spectrum. In principle, the string doublet–triplet splitting mechanism operates in all the models that include the symmetry breaking pattern $SO(10) \rightarrow SO(6) \times SO(4)$. In the PS models, however, the Higgs representations that induce $SU(4) \times SU(2)_R \rightarrow SU(3)_C \times U(1)_Y$ contain the Higgs triplet representations with the quantum numbers of eq. (2.3), that may mediate rapid proton decay through dimension five operators. In the supersymmetric PS models the color triplets in the vectorial representation $(6, 1, 1)$ are used to give large mass to the Higgs color triplets, by the superpotential terms $\lambda_2 H H D + \lambda_3 \bar{H} \bar{H} \bar{D}$, when the fields H and \bar{H} develop a large VEV of the order of the GUT scale. On the other hand, in the SU421 models the Higgs representations of eq. (2.8) do not contain the fields with the quantum numbers of eq. (2.3). Therefore, the string doublet–triplet splitting mechanism is useful only in models with $SU(3)_C \times SU(2)_L \times U(1)^2$, $SU(4)_C \times SU(2)_L \times U(1)_R$, or $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, as the $SO(10)$ subgroup which remains unbroken by the GSO projections.

Another important advantage of the SU421 string models is with respect to the electroweak Higgs doublet representations. In the left–right symmetric models, *i.e.* the PS and LRS models in which $SU(2)_L$ and $SU(2)_R$ both remain unbroken at the string scale, up–quark and down–quark masses both arise from the coupling to a Higgs bi–doublet. This introduces the danger of inducing Flavor Changing Neutral Currents (FCNC) at an unacceptable rate [13]. A possible solution is to use two bi–doublet Higgs representations, one of which is used to give masses to the up–type quarks, while the second is used to give masses to the down–type quarks. This, however, introduces a bi–doublet splitting problem. Namely, we must insure that one Higgs multiplet remains light to give mass to the up– or down–type quarks, while the second Higgs multiplet in the respective bi–doublet becomes sufficiently heavy so as to avoid problems with FCNC. Arguably, this can be achieved in a field theory setting. However, the bi–doublet splitting mechanisms that have been discussed in the literature [14] utilize $SU(2)$ triplet representations that are, in general, not present in the free fermionic string models. Therefore, whether or not bi–doublet splitting can be achieved in the left–right symmetric string models is an open question. By contrast, in the SU421 models $SU(2)_R$ is broken at the string level. Consequently, the bi–doublet Higgs is split already at the string level. The up and down quarks therefore couple to separate doublet Higgs multiplets, and the problem with FCNC is evaded.

The SU421 models should also contain four additional singlet fields ϕ_0 and $\phi_{i=1,2,3}$. ϕ_0 acquires a VEV of the order of the electroweak scale which induces the electroweak Higgs doublet mixing, while ϕ_i are used to construct an extended see–saw mechanism that generates light Majorana masses for the left–handed neutrinos. The tree level superpotential of the model is given by:

$$W = \lambda_{ij}^1 F_L^i U_R^j \bar{h} + \lambda_{ij}^2 F_L^i D_R^j h + \lambda_{ij}^3 U_R^i \bar{H} \phi^j + \lambda^4 \bar{h} h \phi^0 + \lambda_5 \Phi^3, \quad (2.9)$$

where $\Phi = \{\phi^i, \phi^0\}$. The superpotential in eq. (2.9) leads to the neutrino mass matrix

$$\begin{pmatrix} 0 & m_u^{ij} & 0 \\ m_u^{ji} & 0 & \langle \bar{H} \rangle \\ 0 & \langle \bar{H} \rangle & \langle \phi_0 \rangle \end{pmatrix}, \quad (2.10)$$

whose diagonalization gives three light neutrinos with masses of the order $\langle \phi_0 \rangle (m_u^{ij} / \langle \bar{H} \rangle)^2$ and gives heavy mass, of order $\langle \bar{H} \rangle$, to the right-handed neutrinos.

Below the scale of $SU(4)_C \times U(1)_R$ breaking the SU421 models should reproduce the spectrum and couplings of the MSSM. As we have seen SU421 string models offer important advantages with respect to the doublet-triplet and bi-doublet splitting problems. A field theory analysis of these models is therefore of further interest. As our interest here is primarily in the string construction of SU421 models we do not enter into further field theory details. We emphasize that our intent here is not to construct a fully realistic SU421 model, but merely to study the structure of the NAHE-based free fermionic string models with this choice of the $SO(10)$ subgroup. In this respect we note that the bi-doublet splitting problem introduces further motivation for the choice of $SU(3) \times SU(2) \times U(1)^2$ or $SU(4) \times SU(2) \times U(1)$ as the $SO(10)$ subgroup that remain unbroken after application of the string GSO projections. Thus, while the doublet-triplet splitting problem does not distinguish between the PS string model ($SO(10) \rightarrow SO(6) \times SO(4)$), or LRS string model ($SO(10) \rightarrow SU(3) \times SU(2)^2 \times U(1)$), and the SLM string model ($SO(10) \rightarrow SU(3) \times SU(2) \times U(1)^2$) or SU421 string model ($SO(10) \rightarrow SU(4) \times SU(2) \times U(1)$), the bi-doublet splitting problem favors the later choices. The SLM and SU421 string models provide a stringy solution both to the doublet-triplet splitting problem, as well as to the bi-doublet splitting problem. However, as we discuss below the NAHE-based free fermionic models do not produce viable SU421 string models.

3 SU421 free fermionic models

A model in the free fermionic formulation [15] is constructed by choosing a consistent set of boundary condition basis vectors. The basis vectors, \mathbf{b}_k , span a finite additive group $\Xi = \sum_k n_k \mathbf{b}_k$ where $n_k = 0, \dots, N_{z_k} - 1$. The physical massless states in the Hilbert space of a given sector $\alpha \in \Xi$, are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections. The $U(1)$ charges, $Q(f)$, for the unbroken Cartan generators of the four dimensional gauge group are in one to one correspondence with the $U(1)$ currents f^*f for each complex fermion f , and are given by:

$$Q(f) = \frac{1}{2}\alpha(f) + F(f), \quad (3.1)$$

where $\alpha(f)$ is the boundary condition of the world-sheet fermion f in the sector α , and $F_\alpha(f)$ is a fermion number operator counting each mode of f once (and if f is

complex, f^* minus once). For periodic fermions, $\alpha(f) = 1$, the vacuum is a spinor representation of the Clifford algebra of the corresponding zero modes. For each periodic complex fermion f there are two degenerate vacua $|+\rangle, |-\rangle$, annihilated by the zero modes f_0 and f_0^* and with fermion numbers $F(f) = 0, -1$, respectively.

The realistic models in the free fermionic formulation are generated by a basis of boundary condition vectors for all world-sheet fermions [2, 3, 5, 4, 18]. The basis is constructed in two stages. The first stage consists of the NAHE set [2, 12, 4]*, which is a set of five boundary condition basis vectors, $\{\mathbf{1}, \mathbf{S}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$. The gauge group after the NAHE set is $SO(10) \times SO(6)^3 \times E_8$ with $N = 1$ space-time supersymmetry. The vector \mathbf{S} is the supersymmetry generator and the superpartners of the states from a given sector α are obtained from the sector $\mathbf{S} + \alpha$. The space-time vector bosons that generate the gauge group arise from the Neveu-Schwarz (NS) sector and from the sector $\zeta \equiv \mathbf{1} + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$. The NS sector produces the generators of $SO(10) \times SO(6)^3 \times SO(16)$. The sector ζ produces the spinorial **128** of $SO(16)$ and completes the hidden gauge group to E_8 . The vectors $\mathbf{b}_1, \mathbf{b}_2$ and \mathbf{b}_3 produce 48 spinorial **16**'s of $SO(10)$, sixteen from each sector $\mathbf{b}_1, \mathbf{b}_2$ and \mathbf{b}_3 . The vacuum of these sectors contains eight periodic worldsheet fermions, five of which produce the charges under the $SO(10)$ group, while the remaining three periodic fermions generate charges with respect to the flavor symmetries. Each of the sectors $\mathbf{b}_1, \mathbf{b}_2$ and \mathbf{b}_3 is charged with respect to a different set of flavor quantum numbers, $SO(6)_{1,2,3}$.

The NAHE set divides the 44 right-moving and 20 left-moving real internal fermions in the following way: $\bar{\psi}^{1,\dots,5}$ are complex and produce the observable $SO(10)$ symmetry; $\bar{\phi}^{1,\dots,8}$ are complex and produce the hidden E_8 gauge group; $\{\bar{\eta}^1, \bar{y}^{3,\dots,6}\}, \{\bar{\eta}^2, \bar{y}^{1,2}, \bar{\omega}^{5,6}\}, \{\bar{\eta}^3, \bar{\omega}^{1,\dots,4}\}$ give rise to the three horizontal $SO(6)$ symmetries. The left-moving $\{y, \omega\}$ states are also divided into the sets $\{y^{3,\dots,6}\}, \{y^{1,2}, \omega^{5,6}\}, \{\omega^{1,\dots,4}\}$. The left-moving $\chi^{12}, \chi^{34}, \chi^{56}$ states carry the supersymmetry charges. Each sector $\mathbf{b}_1, \mathbf{b}_2$ and \mathbf{b}_3 carries periodic boundary conditions under $(\psi^\mu | \bar{\psi}^{1,\dots,5})$ and one of the three groups: $(\chi_{12}, \{y^{3,\dots,6} | \bar{y}^{3,\dots,6}\}, \bar{\eta}^1)$, $(\chi_{34}, \{y^{1,2}, \omega^{5,6} | \bar{y}^{1,2} \bar{\omega}^{5,6}\}, \bar{\eta}^2)$, $(\chi_{56}, \{\omega^{1,\dots,4} | \bar{\omega}^{1,\dots,4}\}, \bar{\eta}^3)$.

The second stage of the basis construction consist of adding three additional basis vectors to the NAHE set. Three additional vectors are needed to reduce the number of generations to three, one from each sector $\mathbf{b}_1, \mathbf{b}_2$ and \mathbf{b}_3 . One specific example is given in Table (4.1). The choice of boundary conditions to the set of real internal fermions $\{y, \omega | \bar{y}, \bar{\omega}\}^{1,\dots,6}$ determines the low energy properties, such as the number of generations, Higgs doublet-triplet splitting and Yukawa couplings.

The $SO(10)$ gauge group is broken to one of its subgroups $SU(5) \times U(1)$, $SO(6) \times SO(4)$ or $SU(3) \times SU(2) \times U(1)^2$ by the assignment of boundary conditions to the set $\bar{\psi}_{\frac{1}{2}}^{1,\dots,5}$:

*The NAHE-set was first constructed by Nanopoulos, Antoniadis, Hagelin and Ellis (NAHE) as a subset of the flipped $SU(5)$ string model[2]. Its properties and importance for the phenomenological viability of the free fermionic models were discussed in reference [12, 4]. *nahe*=pretty, in Hebrew.

$$1. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \Rightarrow SU(5) \times U(1), \quad (3.2)$$

$$2. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \{11100\} \Rightarrow SO(6) \times SO(4). \quad (3.3)$$

To break the $SO(10)$ symmetry to $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L^\dagger$ both steps, 1 and 2, are used, in two separate basis vectors. The breaking pattern $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is achieved by the following assignment in two separate basis vectors

$$1. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \{11100\} \Rightarrow SO(6) \times SO(4), \quad (3.4)$$

$$2. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}00\} \Rightarrow SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R. \quad (3.5)$$

Similarly, the breaking pattern $SO(10) \rightarrow SU(4)_C \times SU(2)_L \times U(1)_R$ is achieved by the following assignment in two separate basis vectors

$$1. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \{11100\} \Rightarrow SO(6) \times SO(4), \quad (3.6)$$

$$2. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \{000\frac{1}{2}\frac{1}{2}\} \Rightarrow SU(4)_C \times SU(2)_L \times U(1)_R. \quad (3.7)$$

We comment here that a recurring feature of some of the three generation free fermionic heterotic string models is the emergence of a combination of the basis vectors which extend the NAHE set,

$$\mathbf{X} = n_\alpha \alpha + n_\beta \beta + n_\gamma \gamma, \quad (3.8)$$

for which $\mathbf{X}_L \cdot \mathbf{X}_L = 0$ and $\mathbf{X}_R \cdot \mathbf{X}_R \neq 0$. Such a combination may produce additional space-time vector bosons, depending on the choice of GSO phases. These additional space-time vector bosons enhance the four dimensional gauge group. This situation is similar to the presence of the combination of the NAHE set basis vectors $\mathbf{1} + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$, which enhances the hidden gauge group, at the level of the NAHE set, from $SO(16)$ to E_8 . In the free fermionic models this type of gauge symmetry enhancement in the observable sector is, in general, family universal and is intimately related to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold structure which underlies the realistic free fermionic models. Such enhanced symmetries were shown to forbid proton decay mediating operators to all orders of nonrenormalizable terms [18]. Below we discuss examples of models with and without gauge enhancement.

The SU421 symmetry breaking pattern induced by the boundary condition assignment given in eq. (3.7) has an important distinction from the previous symmetry

$$^\dagger U(1)_C = \frac{3}{2}U(1)_{B-L}; U(1)_L = 2U(1)_{T_{3R}}.$$

breaking patterns. As in the previous cases, since it involves a breaking of an $SO(2n)$ group to $SU(n) \times U(1)$ it contains 1/2 boundary conditions. As discussed above the observable and hidden non-Abelian gauge groups arise from the sets of complex world-sheet fermions $\{\bar{\psi}^{1,\dots,5}\bar{\eta}^{1,\dots,3}\}$ and $\{\bar{\phi}^{1,\dots,8}\}$, respectively. The breaking pattern (3.7) entails an assignment of 1/2 boundary condition to two complex fermions in the observable set, whereas the symmetry breaking patterns in eqs. (3.2,3.5) involve three such assignments. On the other hand, the modular invariance rules [15] for the product $\mathbf{b}_j \cdot \gamma$, where \mathbf{b}_j are the NAHE set basis vectors and γ is the basis vector that contains the 1/2 boundary conditions, enforces that no other complex fermion from the observable set has 1/2 boundary conditions. Additionally, the constraint on the product $\gamma \cdot \gamma$ imposes that either 8 or 12 complex fermions have 1/2 boundary conditions. Since, as we saw, only two can have such boundary conditions from the observable set, it implies that six and only six from the hidden set must have 1/2 boundary conditions. This is in contrast to the other cases that allow assignment of twelve 1/2 boundary conditions in the basis vector γ . The consequence of having only eight 1/2 boundary conditions in the basis vector γ is the appearance of additional sectors that may lead to enhancement of the four dimensional gauge group. Below we discuss several other important distinctions of this symmetry breaking pattern as compared to the previous cases.

4 SU421 model without enhanced symmetry

As our first example of a SU421 free fermionic heterotic string model we consider Model 1, specified below. The boundary conditions of the three basis vectors which extend the NAHE set are shown in Table (4.1). Also given in Table (4.1) are the pairings of left- and right-moving real fermions from the set $\{y, \omega | \bar{y}, \bar{\omega}\}$. These fermions are paired to form either complex, left- or right-moving fermions, or Ising model operators, which combine a real left-moving fermion with a real right-moving fermion. The generalized GSO coefficients determining the physical massless states of Model 1 appear in matrix (4.2).

LRS Model 1 Boundary Conditions:

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	0 0 0 $\frac{1}{2}$ $\frac{1}{2}$	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0

	$y^3 y^6$	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1 \omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	$\bar{y}^1 \bar{\omega}^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$\bar{\omega}^2 \bar{\omega}^4$
α	1	1	1	0	1	1	1	0	1	1	1	0
β	0	1	0	1	0	1	0	1	1	0	0	0
γ	0	0	1	1	1	0	0	0	0	1	0	0

(4.1)

LRS Model 1 Generalized GSO Coefficients:

	1	S	b₁	b₂	b₃	α	β	γ
1	1	1	-1	-1	-1	1	1	1
S	1	1	1	1	1	-1	-1	-1
b₁	-1	-1	-1	-1	-1	-1	-1	1
b₂	-1	-1	-1	-1	-1	1	1	1
b₃	-1	-1	-1	-1	-1	-1	1	i
α	1	-1	1	-1	1	1	1	i
β	1	-1	-1	1	-1	-1	-1	i
γ	1	-1	-1	1	-1	-1	1	i

(4.2)

In matrix (4.2) only the entries above the diagonal are independent and those below and on the diagonal are fixed by the modular invariance constraints. Blank lines are inserted to emphasize the division of the free phases between the different sectors of the realistic free fermionic models. Thus, the first two lines involve only the GSO phases of $c(\{\mathbf{1}, \mathbf{S}\}_{\mathbf{a}_i})$. The set $\{\mathbf{1}, \mathbf{S}\}$ generates the $N = 4$ model with \mathbf{S} being the space-time supersymmetry generator and therefore the phases $c(\frac{\mathbf{S}}{\mathbf{a}_i})$ are those that control the space-time supersymmetry in the superstring models. Similarly, in the free fermionic models, sectors with periodic and anti-periodic boundary conditions, of the form of \mathbf{b}_i , produce the chiral generations. The phases $c(\frac{\mathbf{b}_i}{\mathbf{b}_j})$ determine the chirality of the states from these sectors.

In the free fermionic models the basis vectors \mathbf{b}_i are those that respect the $SO(10)$ symmetry while the vectors denoted by Greek letters are those that break the $SO(10)$ symmetry. As the Standard Model matter states arise from sectors which preserve the $SO(10)$ symmetry, the phases that fix the Standard Model charges are, in general, the phases $c(\frac{\mathbf{b}_i}{\mathbf{a}_i})$. On the other hand, the basis vectors of the form $\{\alpha, \beta, \gamma\}$ break the $SO(10)$ symmetry. The phases associated with these basis vectors are associated with exotic physics, beyond the Standard Model. These phases, therefore, also affect the final four dimensional gauge symmetry.

The final gauge group in Model 1 arises as follows: In the observable sector the NS boundary conditions produce gauge group generators for

$$SU(4)_C \times SU(2)_L \times U(1)_R \times U(1)_{1,2} \times SU(2)_3 \times U(1)_{4,5} \times SU(2)_6. \quad (4.3)$$

Here the flavor $SU(2)_{3,6}$ symmetries are generated by $\{\bar{\eta}^3 \bar{\zeta}^3\}$ where $\bar{\zeta}^3 = 1/\sqrt{2}(\bar{w}^2 + i\bar{w}^4)$. In previous free fermionic models this group factor breaks to $U(1)^2$, but this is

an artifact of the specific model considered in eq. (4.1), and is not a generic feature of SU421 models. Thus, the $SO(10)$ symmetry is broken to $SU(4) \times SU(2)_L \times U(1)_R$, as discussed above, where,

$$U(1)_R = \text{Tr } U(2)_L \Rightarrow Q_R = \sum_{i=4}^5 Q(\bar{\psi}^i). \quad (4.4)$$

The flavor $SO(6)^3$ symmetries are broken to $U(1)_{1,2} \times SU(2)_3 \times U(1)_{4,5} \times SU(2)_6$. In the hidden sector the NS boundary conditions produce the generators of

$$SU(4)_H \times SU(2)_{H1} \times SU(2)_{H2} \times SU(2)_{H3} \times U(1)_{7,8} \quad (4.5)$$

where $SU(2)_{H1,H2}$ and $SU(2)_{H3}$ arise from the complex world-sheet fermions $\{\bar{\phi}^7 \bar{\phi}^8\}$ and $\{\bar{\phi}^5 \bar{\phi}^6\}$, respectively; and $U(1)_7$ and $U(1)_8$ correspond to the combinations of world-sheet charges

$$Q_7 = \sum_{i=5}^6 Q(\bar{\phi})^i. \quad (4.6)$$

$$Q_8 = \sum_{i=1}^4 Q(\bar{\phi})^i, \quad (4.7)$$

As we discussed in section (3) the SU421 models contain additional sectors that may produce space-time vector bosons and enhance the four dimensional gauge group. In the model of eq. (4.1) these are the sectors 2γ , $\zeta_1 \equiv 1 + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$ and $\zeta_2 \equiv 1 + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3 + 2\gamma$. However, due to the choice of one-loop phases in eq. (4.2) all the additional vector bosons from these sectors are projected out by the GSO projections and there is therefore no gauge enhancement from these sectors in this model. These sectors are generic in the SU421 models.

In addition to the graviton, dilaton, antisymmetric sector and spin-1 gauge bosons, the NS sector gives two pairs of color triplets, transforming as $(6,1,0)$ under $SU(4)_C \times SU(2)_L \times U(1)_R$; three quadruplets of $SO(10)$ singlets with $U(1)_{1,2,3}$ charges; and three singlets of the entire four dimensional gauge group. The states from the sectors \mathbf{b}_j ($j = 1, 2, 3$) produce the three light twisted generations. These states and their decomposition under the entire gauge group are shown in Appendix A. The remaining massless states and their quantum numbers also appear in Appendix A.

A distinct feature of the SU421 NAHE-based free fermionic models is with respect to the states from the twisted sectors $\mathbf{b}_{1,2,3}$. As discussed in section (3), in the NAHE-based free fermionic models the twisted sectors \mathbf{b}_j produce the three chiral 16 of $SO(10)$ decomposed under the final $SO(10)$ subgroup. In the case of the FSU5, PS and SLM models the 2γ projection fixes the sign of the charge under $U(1)_{1,2,3}$ to be either plus or minus. The consequence is that in these models there exists a combination of the flavor symmetries, which is anomalous. Additionally, in these models the 2γ projection does not break the $SO(10)$ symmetry. By contrast, in the

left–right symmetric models of ref. [6], and in the SU421 models, the 2γ projection breaks the $SO(10)$ symmetry. In the FSU5, PS, SLM and LRS models, however, the 2γ projection selects complete 16 representations of $SO(10)$ that carry different charges under the flavor $U(1)$ symmetries, *i.e.* in the orbifold language they may attach to different fixed points. By contrast, however, in the case of the SU421 models the 2γ projection selects partial fillings of the 16 representation of $SO(10)$, *i.e.* either the left or right–handed fields. The consequence is that the SU421 models do not produce three complete Standard Model generations and hence the models are not realistic.

As discussed above the existence of an anomalous $U(1)$ symmetry is a general outcome of free fermionic models. However, in the case of LRS free fermionic models there exist three chiral generation models that are completely free of Abelian and non–Abelian anomalies. This is again a consequence of the 2γ projection. In the case of the FSU5, PS and SLM models the 2γ vector breaks the $E_8 \times E_8$ gauge group to $SO(16) \times SO(16)$. The result of the $Z_2 \times Z_2$ orbifold projection is then to break the observable $SO(16)$ gauge symmetry to $SO(10) \times U(1)^3$. Alternatively, we can view the $Z_2 \times Z_2$ projection as breaking $E_8 \rightarrow E_6 \times U(1)^2$, and the 2γ projection induces the breaking $E_6 \rightarrow SO(10) \times U(1)_A$, where $U(1)_A$ is the anomalous $U(1)$ combination. However, the LRS free fermionic string models do not start with the $N = 4$ $E_8 \times E_8$ or $SO(16) \times SO(16)$ vacua. Rather, in this case the starting $N = 4$ vacua has $SO(16) \times E_7 \times E_7$ gauge group. The $Z_2 \times Z_2$ orbifold acts as in the previous models. The appearance of an anomalous $U(1)$ symmetry in the FSU5, PS and SLM string models is therefore tied to the breaking of $E_6 \rightarrow SO(10) \times U(1)_A$, and arises from the chiral spectrum. In the LRS string models the charges of the chiral spectrum under $U(1)_A$ cancels sector by sector and therefore the $U(1)_{1,2,3}$ are anomaly free and are absent from the anomalous $U(1)$ combination. On the other hand the additional $U(1)$ symmetries that arise from the Narain lattice can be anomalous in the LRS models and indeed LRS string models that do contain an anomalous $U(1)$ were presented in ref. [6, 11]. As we demonstrate below the SU421 models give rise to anomaly free models as well as to models that contain an anomalous $U(1)$ symmetry. However, in contrast to the LRS symmetric models the cancellation of the anomalous $U(1)$ charges of the chiral spectrum may occur between different sectors rather than sector by sector.

5 Models with enhanced non–Abelian symmetries

We next turn to our second example, Model 2. The boundary condition basis vectors and one–loop phases, which define the model, are given in Table (5.1) and matrix (5.2), respectively.

LRS Model 2 Boundary Conditions:

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	0 0 0 $\frac{1}{2}$ $\frac{1}{2}$	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0

	$y^3 y^6$	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1 \omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	$\bar{y}^1 \bar{\omega}^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$\bar{\omega}^2 \bar{\omega}^4$
α	0	0	1	1	1	0	0	0	0	1	0	1
β	1	0	0	0	0	0	1	1	0	0	1	1
γ	0	1	0	1	0	1	0	1	1	0	0	1

(5.1)

LRS Model 2 Generalized GSO Coefficients:

	1	S	b₁	b₂	b₃	α	β	γ
1	1	1	-1	-1	-1	1	1	1
S	1	1	1	1	1	-1	-1	-1
b₁	-1	-1	-1	-1	-1	1	-1	-1
b₂	-1	-1	-1	-1	-1	-1	-1	-1
b₃	-1	-1	-1	-1	-1	1	1	i
α	1	-1	1	1	1	-1	1	-1
β	1	-1	1	-1	1	1	-1	1
γ	-1	-1	1	1	-1	-1	1	-1

(5.2)

The total gauge group of Model 2 arises as follows. In the observable sector the NS boundary conditions produce the generators of $SU(4)_C \times SU(2)_L \times U(1)_R \in SO(10) \times U(1)_{1,2,3} \times U(1)_{4,5,6}$, while in the hidden sector the NS boundary conditions produce the generators of

$$SU(4)_H \times SU(2)_{H_1} \times SU(2)_{H_2} \times SU(2)_{H_3} \times U(1)_7 \times U(1)_8. \quad (5.3)$$

$U(1)_7$ and $U(1)_8$ correspond to the combinations of the world-sheet charges given in eqs. (4.7) and (4.6), respectively.

Model 2 contains two combinations of non-NAHE basis vectors with $\mathbf{X}_L \cdot \mathbf{X}_L = 0$, which therefore may give rise to additional space-time vector bosons. The first is the sector 2γ . The second arises from the vector combination given by $\zeta + 2\gamma$, where $\zeta \equiv 1 + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$. Both sectors arise from the NAHE set basis vectors plus 2γ and are therefore independent of the assignment of periodic boundary conditions in the basis vectors α , β and γ . Both are therefore generic for the pattern of symmetry breaking $SO(10) \rightarrow SU(4)_C \times SU(2)_L \times U(1)_R$, in NAHE based models.

In Model 1 the additional space–time vector bosons from both sectors are projected out and therefore there is no gauge enhancement. In Model 2 all the space–time vector bosons from the sector 2γ are projected out by the GSO projections and therefore give no gauge enhancement from this sector. The sector $\zeta + 2\gamma$ may, or may not, give rise to additional space–time vector bosons, depending on the choice of GSO phase

$$c\left(\begin{smallmatrix} \gamma \\ \mathbf{b}_3 \end{smallmatrix}\right) = \pm 1, \quad (5.4)$$

where with the $+1$ choice all the additional vector bosons are projected out, whereas the -1 choice gives rise to additional space–time gauge bosons which are charged with respect to the $SU(2)_L \times SU(2)_{H_1}$ groups. This enhances the $SU(2)_L \times SU(2)_{H_1}$ group to $SO(5)$. Thus, in this case, the full massless spectrum transforms under the final gauge group, $SU(4)_C \times SO(5) \times U(1)_R \times U(1)_{1,2,3} \times U(1)_{4,5,6} \times SU(4)_H \times SU(2)_{H_2} \times SU(2)_{H_3} \times U(1)_{7,8}$.

In addition to the graviton, dilaton, antisymmetric sector and spin–1 gauge bosons, the NS sector gives rise to three quadruplets of $SO(10)$ singlets with $U(1)_{1,2,3}$ charges; and three singlets of the entire four dimensional gauge group. The states from the sectors $\mathbf{b}_j \oplus \zeta + 2\gamma$ ($j = 1, 2, 3$) produce the three light generations. The states from these sectors and their decomposition under the entire gauge group are shown in Appendix B.

6 Anomalous $U(1)$

A general property of the realistic free fermionic heterotic string models, which is also shared by many other superstring vacua, is the existence of an “anomalous” $U(1)$. The presence of an Abelian anomalous symmetry in superstring derived models yields many desirable phenomenological consequences from the point of view of the effective low energy field theory. Indeed, the existence of such an anomalous $U(1)$ symmetry in string derived models has inspired vigorous attempts to understand numerous issues, relevant for the observable phenomenology, including: the fermion mass spectrum, supersymmetry breaking cosmological implications, and more. From the perspective of string phenomenology an important function of the anomalous $U(1)$ is to induce breaking and rank reduction of the four dimensional gauge group. In general, the existence of an anomalous $U(1)$ in a string model implies that the string vacuum is unstable and must be shifted to a stable point in the moduli space. This arises because, by the Green–Schwarz anomaly cancellation mechanism, the anomalous $U(1)$ gives rise to a Fayet–Iliopoulos term which breaks supersymmetry. Supersymmetry is restored and the vacuum is stabilized by sliding the vacuum along flat F and D directions. This is achieved by assigning non–vanishing VEVs to some scalar fields in the massless string spectrum.

An important issue in string phenomenology is therefore to understand what are

the general conditions for the appearance of an anomalous $U(1)$ and under what conditions an anomalous $U(1)$ is absent. The previously studied realistic free fermionic string models that include the FSU5, PS, and SLM types, have always contained an anomalous $U(1)$ symmetry. In contrast, in ref. [6] it was shown that the LRS models also give rise to vacua in which all the $U(1)$ symmetries in the four dimensional gauge group are anomaly free, as well as to vacua that do contain an anomalous $U(1)$ symmetry. The distinction between the different cases, and the properties of the models that resulted in the presence, or the absence, of an anomalous $U(1)$ symmetry were discussed in ref. [6]. Next we examine the presence of an anomalous $U(1)$ symmetry in the SU421 string models.

For completeness we first discuss the case of the previously studied free fermionic models, *i.e.* the FSU5, the PS, the SLM and the LRS string models. The question of the anomalous $U(1)$ symmetry in string models, in general, and in the free fermionic models, in particular, was studied in some detail in ref. [16, 17]. The anomalous $U(1)$ in the free fermionic models is in general a combination of two distinct kinds of world-sheet $U(1)$ currents, those generated by $\bar{\eta}^j$ and those generated by the additional complexified fermions from the set $\{\bar{y}, \bar{\omega}\}^{1,\dots,6}$. The trace of the $U(1)$ charges of the entire massless string spectrum can then be non-vanishing under some of these world-sheet $U(1)$ currents. One combination of these $U(1)$ currents then becomes the anomalous $U(1)$, whereas all the orthogonal combinations are anomaly free. To understand the origin of the anomalous $U(1)$ in the realistic free fermionic models, it is instructive to consider the contributions from the two types of world-sheet $U(1)$ currents separately.

In ref. [17] it was shown that the anomalous $U(1)$ in the realistic free fermionic models can be seen to arise due to the breaking of the world-sheet supersymmetry from (2,2) to (2,0). Consider the set of boundary condition basis vectors $\{\mathbf{1}, \mathbf{S}, \zeta, \mathbf{X}, \mathbf{b}_1, \mathbf{b}_2\}$ [17], which produces (for an appropriate choice of the GSO phases) the model with $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ gauge group. It was shown that if we choose the GSO phases such that $E_6 \rightarrow SO(10) \times U(1)$, the $U(1)$ in the decomposition of E_6 under $SO(10) \times U(1)$ becomes the anomalous $U(1)$. This $U(1)$ is produced by the combination of world-sheet currents $\bar{\eta}^1 \bar{\eta}^{1*} + \bar{\eta}^2 \bar{\eta}^{2*} + \bar{\eta}^3 \bar{\eta}^{3*}$. We can view all of the realistic FSU5, PS, and SLM free fermionic string models as being related to this $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ string vacuum. This combination of $U(1)$ currents therefore contributes to the anomalous $U(1)$ in all the realistic free fermionic models with FSU5, PS, or SLM gauge groups.

The existence of the anomalous $U(1)$ in the FSU5, PS, or SLM, and its absence in the LRS string models can be traced to different $N = 4$ string vacua in four dimensions. While in the E_6 model one starts with an $N = 4$ $SO(12) \times E_8 \times E_8$ string vacua, produced by the set $\{\mathbf{1}, \mathbf{S}, \mathbf{X}, \zeta\}$ [17], we can view the FSU5, PS, and SLM string models as starting from an $N = 4$ $SO(12) \times SO(16) \times SO(16)$ string vacua. In this case the two spinorial representations from the sectors \mathbf{X} and ζ , that complete the adjoint of $SO(16) \times SO(16)$ to $E_8 \times E_8$, are projected out by the choice

of the GSO projection phases. The subsequent projections, induced by the basis vectors \mathbf{b}_1 and \mathbf{b}_2 , which correspond to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold twistings, then operate identically in the two models, producing in one case the E_6 , and in the second case the $SO(10) \times U(1)$, gauge groups, respectively. The important point, however, is that both cases preserve the “standard embedding” structure which splits the observable and hidden sectors. The important set in this respect is the set $\{\mathbf{1}, \mathbf{S}, \mathbf{X}, \zeta\}$, where \mathbf{X} has periodic boundary conditions for $\{\bar{\psi}^1, \dots, \bar{\psi}^5, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\}$. The choice of the phase $c_{\zeta}^{\mathbf{X}} = \pm 1$ fixes the vacuum to $E_8 \times E_8$ or $SO(16) \times SO(16)$.

In contrast, the LRS free fermionic string models do not start with the $N = 4$ $E_8 \times E_8$ or $SO(16) \times SO(16)$ vacua. Rather, in this case the starting $N = 4$ vacua can be seen to arise from the set of boundary condition basis vectors $\{\mathbf{1}, \mathbf{S}, 2\gamma, \zeta\}$. Starting with this set and with the choice of GSO projection phases

$$\begin{array}{c} \mathbf{1} \quad \mathbf{S} \quad \zeta \quad 2\gamma \\ \mathbf{1} \left(\begin{array}{cccc} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{array} \right), \end{array} \quad (6.1)$$

the resulting string vacua has $N = 4$ space-time supersymmetry with $SO(16) \times E_7 \times E_7$ gauge group. The sectors \mathbf{b}_1 and \mathbf{b}_2 are then added as in the previous models. The LRS string models therefore do not preserve the “standard embedding” splitting between the observable and hidden sectors. This is the first basic difference between the FSU5, PS, or SLM, and the LRS free fermionic models.

Now turn to the case of the three generation models. The chirality of the generations from the sectors \mathbf{b}_j ($j = 1, 2, 3$) is induced by the projection which breaks $N = 2 \rightarrow N = 1$ space-time supersymmetry. Chirality for the generations is therefore fixed by the GSO projection phase $c_{\mathbf{b}_j}^{\mathbf{b}_i}$ with $i \neq j$. On the other hand, generation charges under $U(1)_j$ are fixed by the \mathbf{X} projection in the E_6 model, by the projection induced by the vector 2γ of the FSU5, PS, and SLM string models, or by the vector 2γ of the LRS string models. The difference is that in the case of the FSU5, PS, and SLM string models the 2γ projection fixes the same sign for the $U(1)_j$ charges of the states from the sectors \mathbf{b}_j . In contrast, in the LRS free fermionic models the corresponding 2γ projection fixes one sign for the $(Q_R + L_R)_j$ states and the opposite sign for the $(Q_L + L_L)_j$ states. The consequence is that the total trace vanishes and the sectors \mathbf{b}_j do not contribute to the trace of the $U(1)_j$ charges. This is in fact the reason that LRS free fermionic models can appear without an anomalous $U(1)$.

The existence of LRS free fermionic string models without an anomalous $U(1)$ does not preclude the possibility of other LRS models with an anomalous $U(1)$. Model 3 of ref. [6] contains three anomalous $U(1)$ symmetries, of which one combination,

$$U(1)_A = U_4 + U_5 + U_6, \quad (6.2)$$

is anomalous, while two orthogonal combinations are anomaly free. In this model the anomalous $U(1)$'s correspond to $U(1)$ symmetries which arise from the additional complexified world-sheet fermions in the set $\{\bar{y}, \bar{\omega}\}^{1,\dots,6}$. This is in agreement with the observation that the $U(1)_{j=1,2,3}$, which are generated by the $\bar{\eta}^j$ world-sheet fermions, are anomaly free in the LRS free fermionic string models.

In the case of the SU421 models the starting $N = 4$ vacua differs again from the previous cases. In this case the set of boundary condition basis vectors $\{\mathbf{1}, \mathbf{S}, 2\gamma, \zeta\}$, with γ being that of eqs. (4.1, 5.1) is to produce the gauge group $SO(28) \times E_8$. In this model the NS sector produces the generators of the adjoint of $SO(24) \times SU(2) \times SU(2) \times SO(12) \times SU(2) \times SU(2)$; the sectors ζ and 2γ produce gauge bosons that transform in the $(1, 1, 2, 32, 1, 1)$ and $(1, 1, 1, \overline{32}, 2, 1)$ representations of the NS gauge group; and the sector $\zeta + 2\gamma$ produces gauge boson that transform as $(24, 2, 1, 1, 1, 2)$ and $(1, 1, 2, 12, 2, 1)$ hence completing the gauge group to $SO(28) \times E_8$. The SU421 models are similar to the LRS models in the sense that, in contrast to the case of the FSU5, PS and SLM models, the periodic fermions in the basis vectors 2γ and ζ are not disjoint. The consequence is that, for either choice of the GSO phase $c\left(\frac{\zeta}{2\gamma}\right)$, the $N = 4$ gauge group is the same. Hence, discrete choice of this phase does not generate an anomalous $U(1)$. Consequently, both the LRS $SO(10)$ subgroup as well as the SU421 one produce models in which all the $U(1)$ currents are anomaly free. Both cases, however, can produce models which do contain an anomalous $U(1)$. However, as discussed above, whereas, in the case of the LRS models the cancellation of the $U(1)_j$ ($j = 1, 2, 3$) is within sectors, in the case of the SU421 models the cancellation may be between sectors. This difference is again traced back to the different $N = 4$ vacua from which the models originate, as both cases share the basic NAHE-set structure. In the model of eq. (4.1) we find that the cancellation is within sectors and there is no anomalous $U(1)$ symmetry. In the model of eq. (5.1) we find that the situation is more intricate. In this case the presence or absence of an anomalous $U(1)$ depends on the GSO phase $c\left(\frac{\mathbf{b}_3}{\gamma}\right) = \pm i$ that also fixes the four dimensional gauge group in this model. We therefore find that the choice $c\left(\frac{\mathbf{b}_3}{\gamma}\right) = -i$ results in an anomaly free model, whereas the choice $c\left(\frac{\mathbf{b}_3}{\gamma}\right) = +i$ results in a model that contains an anomalous $U(1)$ and gauge enhancement from the sector $\zeta + 2\gamma$. This situation is reminiscent of the one in the FSU5, PS and SLM NAHE-based models, in which the presence or absence of an anomalous $U(1)$ symmetry is related to the choice of the four dimensional gauge group. In the anomaly free model we find that the sectors $\mathbf{b}_3 \oplus \mathbf{b}_3 + \zeta + 2\gamma$ contribute with opposite sign to $\text{tr } Q_3 = -8 + 8 = 0$. Similarly, the sectors $S + \mathbf{b}_2 + \mathbf{b}_3 + \alpha + \beta + \gamma \oplus 1 + S + \mathbf{b}_1 + \alpha + \beta + \gamma$ and $S + \mathbf{b}_1 + \mathbf{b}_3 + \alpha + \beta + \gamma \oplus 1 + S + \mathbf{b}_2 + \alpha + \beta + \gamma$ contribute $\text{tr } Q_3 = 4 - 4 = 0$. With $c\left(\frac{\mathbf{b}_3}{\gamma}\right) = +i$ we find that these sectors contribute with equal sign to $\text{tr } Q_3$. In this case the sector $\mathbf{b} + 3 \oplus \mathbf{b}_3 + \zeta + 2\gamma$ together form the complete representations of the enhanced $SO(5)$ gauge group, and give $\text{tr } Q_3 = 8 + 8 = 16$. Similarly, the sectors $S + \mathbf{b}_2 + \mathbf{b}_3 + \alpha + \beta + \gamma \oplus 1 + S + \mathbf{b}_1 + \alpha + \beta + \gamma$ and $S + \mathbf{b}_1 + \mathbf{b}_3 + \alpha + \beta + \gamma \oplus 1 + S + \mathbf{b}_2 + \alpha + \beta + \gamma$

contribute $\text{tr } Q_3 = 12 + 4 = 16$. We therefore see that the $U(1)_3$ gauge symmetry in this model is anomalous with $\text{tr } Q_3 = 48$. As is the case in the FSU5, PS and SLM NAHE-based models the emergence of the anomalous $U(1)$ symmetry is tied to the choice of four dimensional gauge group and the GSO phases. We further note that in the anomaly free model the cancellation is between sectors that belong to the same orbits and that in the anomalous model, due to the choice of GSO phases and breaking of the gauge group, becomes anomalous.

An important issue in the string models that contain an anomalous $U(1)$ symmetry is the existence of supersymmetric flat directions. Since the anomalous $U(1)$ symmetry generates a Fayet–Iliopoulos term at one-loop in string perturbation theory [20] supersymmetry is broken, and is restored by assigning non-vanishing VEVs to massless scalars in the string spectrum along F - and D -flat directions. A vital question is therefore whether such flat directions exist in a given model. The flat directions can be classified generically as Abelian, *i.e.* those that use solely scalars that are singlets of all non-Abelian group factors and non-Abelian flat directions *i.e.* those that also utilize non-Abelian fields. In the case of the FSU5, PS and SLM NAHE-based models Abelian flat directions were always found to exist. In the case of the LRS models on the other hand it was shown in ref. [11] that Abelian flat directions did not exist in model 3 of ref. [6] and only non-Abelian flat directions exist in the model.

In the case of the SU421 non-anomalous model that we study here, Abelian flat directions exist, but are rather trivial. Besides the three uncharged moduli, the only other singlet scalars are two independent vector-like pairs from the NS sector, $(\phi_1, \bar{\phi}_1)$ with $Q_1 = Q_2 = \pm 1$ and $(\phi_2, \bar{\phi}_2)$ with $Q_1 = -Q_2 = \pm 1$. Thus, the elements of the basis set of flat directions are simply $\langle \phi_1 \rangle = \langle \bar{\phi}_1 \rangle$ and $\langle \phi_2 \rangle = \langle \bar{\phi}_2 \rangle$.

In contrast, in the case of the anomalous SU421 model, the set of Abelian D -flat directions is more complex. In addition to the uncharged moduli, there are 32 non-Abelian singlet scalars that form 16 vector-like pairs, as shown in Appendix B. These 16 vector-like pairs generate an eight element basis set for D -flat directions, as shown in Appendix C. Four basis elements carry -2 anomalous charge (denoted Q_3 in Appendix B) and no charge under the remaining eight non-anomalous Abelian symmetries. Corresponding directions with +2 anomalous charge can be formed from these basis directions by vector-partner field substitution. The remaining four basis directions carry neither anomalous nor non-anomalous Abelian charges. As Appendix B shows, all of the non-Abelian fields in this model carry anomalous charge $Q_3 = 0$ or $+1/2$. Thus, one or more of the the four basis directions carrying -2 anomalous charge must appear in the non-perturbatively chosen D -flat direction (for cancellation of the FI D -term).

Twelve of the 32 singlet fields whose combinations of VEVs yield the D -flat basis elements originate in the NS sector. Four of these twelve NS fields carry -1 anomalous charge while their four corresponding vector partners carry +1 anomalous charge. The remaining two pairs of NS vector-like pairs carry no anomalous charge. The ten

additional vector-like pairs of singlets also lack anomalous charge. Two of these ten pairs originate in the $\mathbf{S} + 2\gamma$ sector and the remaining eight pairs originate in the $\mathbf{b}_2 + \alpha + 2\gamma$ sector. The net contributions from the $\mathbf{S} + 2\gamma$ sector field VEVs is zero for each basis element.

7 $SO(10)$ breaking patterns

In this section we present a general argument why free fermionic models with $SU(4) \times SU(2) \times U(1)$ cannot in fact be constructed. For this purpose let us recall that the weight lattice of the spinorial $SO(10)$ representation is made of an even number of $|-\rangle$ Ramond vacua, out of the total five that make up the $SO(10)$ lattice. The sixteen available states can be represented in combinatorial form

$$\left[\binom{5}{0} + \binom{5}{2} + \binom{5}{4} \right] \quad (7.1)$$

where the combinatorial factor counts the number of $|-\rangle$ states in the vacuum. Under the breaking pattern $SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R$ these decompose as

$$Q_R \equiv \left[\binom{3}{0} + \binom{3}{2} \right] \left[\binom{2}{0} + \binom{2}{2} \right] = (\bar{4}, 1, 2) = (\bar{4}, 1, 1)_{+1} + (\bar{4}, 1, 1)_{-1} \quad (7.2)$$

$$Q_L \equiv \left[\binom{3}{1} + \binom{3}{3} \right] \left[\binom{2}{1} \right] = (4, 2, 1) \quad (7.3)$$

where the last step in (7.2) is the decomposition under $SU(4) \times SU(2)_L \times U(1)_L$. We recall that the GSO projection condition on states from a given sector $\alpha \in \Xi$ has the general form [15]

$$\left\{ e^{i\pi(b_i F_\alpha)} - \delta_\alpha c^* \binom{\alpha}{b_i} \right\} |s\rangle = 0 \quad (7.4)$$

with

$$(b_i F_\alpha) \equiv \left\{ \sum_{\substack{real+complex \\ left}} - \sum_{\substack{real+complex \\ right}} \right\} (b_i(f) F_\alpha(f)), \quad (7.5)$$

where $F_\alpha(f)$ is a fermion number operator counting each mode of f once (and if f is complex, f^* minus once). For periodic complex fermions the fermion number of the two degenerate vacua $|+\rangle$, $|-\rangle$ is $F(f) = 0, -1$ respectively. In Eq. (7.4), $\delta_\alpha = -1$ if ψ^μ is periodic in the sector α , and $\delta_\alpha = +1$ if ψ^μ is antiperiodic in the sector α .

As can be seen from eq. (7.4), the assignment of rational boundary conditions in the basis vector that breaks $SU(4) \times SO(4)$ to $SU(4) \times SU(2) \times U(1)$ results in either Q_L or Q_R being selected by the GSO projections. In the free fermionic models that utilize the Z_4 Wilson line breaking, as is done in the models discussed here, the modular invariance constraint on $\gamma \cdot b_j$ imposes that $\gamma\{\bar{\eta}^1; \bar{\eta}^2; \bar{\eta}^3\} \neq 1/2$. Therefore, unlike the situation in the $SU(3) \times U(1) \times SU(2)^2$ models of refs. [6, 11], the result

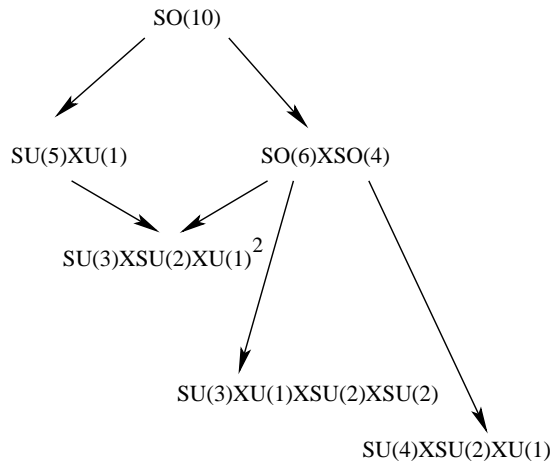


Figure 1: $SO(10)$ symmetry breaking patterns

is that either the Q_L or the Q_R states from a given sector b_j are left invariant by the GSO projections. The basic difference between the $SU(4) \times SU(2) \times U(1)$ model and the $SU(5) \times U(1)$, $SU(3) \times SU(2) \times U(1)^2$ and $SU(3) \times U(1) \times SU(2)^2$ models is that the former uses an even number of rational $1/2$ boundary conditions in γ whereas the later uses an odd number. For this reason the former produces pairs of Q_L or Q_R from the sectors b_j whereas the later produces complete $SO(10)$ multiplets. Now, in order to produce a phenomenologically viable model we need three copies of $(Q_L + Q_R)$. However, it is clear that the SU421 models cannot produce that as, by the argument above, they always produce an even number of this combination. One could contemplate the possibility that the Standard Model representations in such models arise from non-spinorial sectors. However, this will imply that the weak hypercharge is not embedded in $SO(10)$ and therefore $SU(4) \times SU(2) \times U(1)$ will not arise as an $SO(10)$ subgroup. One can contemplate the possibility of using rational boundary conditions other than $1/2$. However, since the problem here arises because of the mismatch of the integral boundary assignment to the set $\{\bar{\psi}^{1,2,3}\}$ versus the rational boundary condition for the remaining pair $\{\bar{\psi}^{4,5}\}$, assigning other rational boundary conditions does not help. Finally, since our argument is solely based on the weight lattice of the spinorial of $SO(10)$ and the GSO projection operator, it is completely general and we expect it to hold in any string construction. Our conclusion is that string models with the symmetry breaking pattern $SO(10) \rightarrow SU(4) \times SU(2) \times U(1)$ are not viable.

The analysis of this paper completes the classification of all the $SO(10)$ subgroups in NAHE based models. The available symmetry breaking patterns are depicted in figure 1. This classification revealed that only the FSU5, PS, SLM, and LRS subgroups produce viable spectra, whereas the SU421 subgroup does not. The cases of LRS and SU421 models also produced models that are completely free of gravitational

and gauge anomalies, whereas the other cases always contain an anomalous $U(1)$. The origin of the differences between the two cases can in fact be traced to the different $N = 4$ vacua from which the models emerge.

8 Conclusions

In this paper we studied the case of NAHE-based free fermionic models with $SU(4) \times SU(2) \times U(1)$ $SO(10)$ subgroup. This case offers some phenomenological advantages as compared to some of the previous case studies, in the sense that it incorporates the elegant string solution to both the doublet-triplet and doublet-doublet splitting problems. We demonstrated, however, that this choice of the $SO(10)$ subgroup as the unbroken four dimensional gauge group of a free fermionic model cannot produce a realistic low energy spectrum.

Additionally, we discussed the important issue of the existence of a anomalous $U(1)$ in the SU421 NAHE-based models. We showed that models which are free of all Abelian and non-Abelian anomalies exist in this class. The absence or presence of an anomalous $U(1)$ is closely tied to the choice of the unbroken four dimensional gauge group, and the ensued cancellation or non-cancellation of the charges in specific orbits. We demonstrated that the anomalous SU421 model, in contrast to the LRS model that was studied in ref. [11], admits Abelian flat directions. This observation is in agreement with the prevailing lore that superstring models generically admit supersymmetric flat directions.

In conclusion we note that to date the NAHE-based models represent the most realistic string models constructed to date. This is particularly evident following the recent neutrino observations that further support the embedding of the Standard Model spectrum in $SO(10)$ representations. As compared to related orbifold models the NAHE-based models possess the distinctive advantage of admitting three generations models with the $SO(10)$ embedding. To advance further our understanding of the potential relevance of string theory to experimental data it is imperative that we delve further in the phenomenological investigations of the most realistic case studies. In this paper we demonstrated that in the NAHE-class a vacuum with three generations and $SU(4) \times SU(2) \times U(1)$ $SO(10)$ subgroup is not viable.

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SEC	$SU(4) \times SU(2)$	Q_R	Q_1	Q_2	Q_4	Q_5	$SU(2)_3 \times SU(2)_6$	$SU(4)_H \times SU(2)_{H_3} \times SU(2)_{H_1} \times SU(2)_{H_2}$	Q_7	Q_8
\mathbf{b}_1	$(4, 1)$	4	-2	0	-2	0	$(1, 1)$	$(1, 1, 1, 1)$	0	0
	$(\bar{4}, 1)$	4	2	0	2	0	$(1, 1)$	$(1, 1, 1, 1)$	0	0
	$(\bar{4}, 1)$	-4	-2	0	-2	0	$(1, 1)$	$(1, 1, 1, 1)$	0	0
	$(\bar{4}, 1)$	-4	2	0	2	0	$(1, 1)$	$(1, 1, 1, 1)$	0	0
\mathbf{b}_2	$(4, 1)$	4	0	-2	0	2	$(1, 1)$	$(1, 1, 1, 1)$	0	0
	$(\bar{4}, 1)$	4	0	2	0	-2	$(1, 1)$	$(1, 1, 1, 1)$	0	0
	$(\bar{4}, 1)$	-4	0	-2	0	2	$(1, 1)$	$(1, 1, 1, 1)$	0	0
	$(\bar{4}, 1)$	-4	0	2	0	-2	$(1, 1)$	$(1, 1, 1, 1)$	0	0
\mathbf{b}_3	$(4, 2)$	0	0	0	0	0	$(1, 2)$	$(1, 1, 1, 1)$	0	0
$S + \mathbf{b}_1 + \mathbf{b}_2 + \alpha + \beta$	$(1, 1)$	0	2	-2	0	0	$(1, 1)$	$(1, 1, 1, 2)$	0	0
	$(1, 1)$	0	-2	2	0	0	$(1, 1)$	$(1, 1, 2, 1)$	0	0
	$(1, 1)$	0	-2	2	0	0	$(1, 1)$	$(1, 1, 1, 2)$	0	0
	$(1, 1)$	0	2	-2	0	0	$(1, 1)$	$(1, 1, 2, 1)$	0	0
$\mathbf{b}_3 + \beta + 2\gamma$	$(1, 1)$	0	0	0	-2	2	$(2, 1)$	$(1, 1, 1, 1)$	4	0
	$(1, 1)$	0	0	0	2	2	$(1, 2)$	$(1, 1, 1, 1)$	4	0
	$(1, 1)$	0	0	0	-2	-2	$(1, 2)$	$(1, 1, 1, 1)$	4	0
	$(1, 1)$	0	0	0	2	-2	$(2, 1)$	$(1, 1, 1, 1)$	4	0
	$(1, 1)$	0	0	0	-2	2	$(2, 1)$	$(1, 1, 1, 1)$	-4	0
	$(1, 1)$	0	0	0	2	2	$(1, 2)$	$(1, 1, 1, 1)$	-4	0
	$(1, 1)$	0	0	0	-2	-2	$(1, 2)$	$(1, 1, 1, 1)$	-4	0
	$(1, 1)$	0	0	0	2	-2	$(2, 1)$	$(1, 1, 1, 1)$	-4	0
$S + 2\gamma$	$(1, 1)$	4	0	0	0	0	$(1, 1)$	$(4, 2, 1, 1)$	0	4
	$(1, 1)$	-4	0	0	0	0	$(1, 1)$	$(\bar{4}, 2, 1, 1)$	0	-4
$1 + S + \mathbf{b}_3 + \alpha + \beta + 2\gamma$	$(1, 2)$	0	2	2	0	0	$(1, 1)$	$(1, 1, 2, 1)$	0	0
	$(1, 2)$	0	-2	-2	0	0	$(1, 1)$	$(1, 1, 2, 1)$	0	0
$1 + \mathbf{b}_1 + \mathbf{b}_2 + 2\gamma$	$(4, 1)$	0	0	0	0	0	$(2, 1)$	$(1, 1, 1, 2)$	0	0
$1 + S + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3 + 2\gamma$	$(1, 2)$	0	0	0	4	0	$(1, 1)$	$(1, 1, 1, 2)$	0	0
	$(1, 2)$	0	0	0	-4	0	$(1, 1)$	$(1, 1, 1, 2)$	0	0

SEC & Field	$SU(4)$ $\times SO(5)$	\mathbb{Q}_R	\mathbb{Q}_1	\mathbb{Q}_2	\mathbb{Q}_3	\mathbb{Q}_4	\mathbb{Q}_5	\mathbb{Q}_6	$SU(4)_H \times SU(2)_{H_3} \times$ $SU(2)_{H_2}$	\mathbb{Q}_7	\mathbb{Q}_8
0:											
$\phi_1, \bar{\phi}_1$	(1,1)	0	0	∓ 1	∓ 1	0	0	0	(1,1,1)	0	0
$\phi_2, \bar{\phi}_2$	(1,1)	0	0	± 1	∓ 1	0	0	0	(1,1,1)	0	0
$\phi_3, \bar{\phi}_3$	(1,1)	0	∓ 1	0	∓ 1	0	0	0	(1,1,1)	0	0
$\phi_4, \bar{\phi}_4$	(1,1)	0	± 1	0	∓ 1	0	0	0	(1,1,1)	0	0
$\phi_5, \bar{\phi}_3$	(1,1)	0	∓ 1	∓ 1	0	0	0	0	(1,1,1)	0	0
ϕ_6, ϕ_6	(1,1)	0	∓ 1	± 1	0	0	0	0	(1,1,1)	0	0
b₁	($\bar{4}$,1)	-4	2	0	0	2	0	0	(1,1,1)	0	0
	($\bar{4}$,1)	4	2	0	0	-2	0	0	(1,1,1)	0	0
	($\bar{4}$,1)	4	-2	0	0	2	0	0	(1,1,1)	0	0
	($\bar{4}$,1)	-4	-2	0	0	-2	0	0	(1,1,1)	0	0
b₂	(4,1)	4	0	-2	0	0	-2	0	(1,1,1)	0	0
	($\bar{4}$,1)	-4	0	-2	0	0	2	0	(1,1,1)	0	0
	($\bar{4}$,1)	-4	0	2	0	0	-2	0	(1,1,1)	0	0
	($\bar{4}$,1)	4	0	2	0	0	2	0	(1,1,1)	0	0
b₃ \oplus b₃ + ζ + 2γ	(4,4)	0	0	0	2	0	0	-2	(1,1,1)	0	0
	(4,4)	0	0	0	2	0	0	2	(1,1,1)	0	0
$S + 2\gamma$:	(1,1)	-4	0	0	0	0	0	0	(6,1,1)	4	0
	(1,1)	4	0	0	0	0	0	0	(6,1,1)	-4	0
	S_1, \bar{S}_1	∓ 4	0	0	0	0	0	0	(1,1,1)	∓ 4	± 8
	S_2, \bar{S}_2	∓ 4	0	0	0	0	0	0	(1,1,1)	∓ 4	∓ 8
b₂ + α + 2γ											
	S_3, \bar{S}_3	0	0	∓ 2	0	∓ 2	∓ 2	∓ 2	(1,1,1)	∓ 4	0
	S_4, \bar{S}_4	0	0	∓ 2	0	∓ 2	± 2	∓ 2	(1,1,1)	± 4	0
	S_5, \bar{S}_5	0	0	∓ 2	0	∓ 2	∓ 2	± 2	(1,1,1)	∓ 4	0
	S_6, \bar{S}_6	0	0	∓ 2	0	∓ 2	± 2	± 2	(1,1,1)	± 4	0
	S_7, \bar{S}_7	0	0	∓ 2	0	± 2	∓ 2	∓ 2	(1,1,1)	± 4	0
	S_8, \bar{S}_8	0	0	∓ 2	0	± 2	± 2	∓ 2	(1,1,1)	∓ 4	0
	S_9, \bar{S}_9	0	0	∓ 2	0	± 2	∓ 2	± 2	(1,1,1)	± 4	0
	S_{10}, \bar{S}_{10}	0	0	∓ 2	0	± 2	± 2	± 2	(1,1,1)	∓ 4	0
$S + \mathbf{b}_2 + \mathbf{b}_3 +$ $\alpha + \beta \pm \gamma$	(1,1)	2	0	2	2	0	-2	0	(4,1,1)	0	0
	(1,1)	2	0	-2	2	0	2	0	(4,1,1)	0	0
	(1,1)	2	0	-2	2	0	2	0	(1,2,1)	-2	0
	(1,1)	2	0	2	2	0	-2	0	(1,2,1)	-2	0
	(1,1)	-2	0	-2	2	0	-2	0	($\bar{4}$,1,1)	0	0
	(1,1)	-2	0	2	2	0	2	0	($\bar{4}$,1,1)	0	0
	(1,1)	-2	0	2	2	0	2	0	(1,2,1)	2	0
	(1,1)	-2	0	-2	2	0	-2	0	(1,2,1)	2	0

$S + \mathbf{b}_1 + \mathbf{b}_3 + \alpha + \beta \pm \gamma$	(1,1)	2	-2	0	2	-2	0	0	(4,1,1)	0	0
	(1,1)	2	2	0	2	2	0	0	(1,2,1)	-2	0
	(1,1)	2	-2	0	2	-2	0	0	(1,2,1)	-2	0
	(1,1)	-2	2	0	2	-2	0	0	(4,1,1)	0	0
	(1,1)	-2	-2	0	2	2	0	0	(4,1,1)	0	0
	(1,1)	-2	-2	0	2	2	0	0	(1,2,1)	2	0
	(1,1)	-2	2	0	2	-2	0	0	(1,2,1)	2	0
	(1,1)	-2	2	0	2	-2	0	0	(1,2,1)	2	0
$S + \mathbf{b}_1 + \mathbf{b}_2 + \alpha + \beta \oplus S + \mathbf{b}_1 + \mathbf{b}_2 + \alpha + \beta + \zeta + 2\gamma$	(1,4)	0	-2	-2	0	0	0	0	(1,1,2)	0	0
	(1,4)	0	2	2	0	0	0	0	(1,1,2)	0	0
$1 + b_1 + \alpha + \beta \pm \gamma$	(1,1)	2	0	2	2	0	-2	0	(1,1,2)	0	0
	(1,1)	2	0	-2	2	0	2	0	(1,1,2)	0	0
	(1,1)	-2	0	-2	2	0	-2	0	(1,1,2)	0	0
	(1,1)	-2	0	2	2	0	2	0	(1,1,2)	0	0
$1 + b_2 + \alpha + \beta \pm \gamma$	(1,1)	2	2	0	2	2	0	0	(1,1,2)	0	0
	(1,1)	2	-2	0	2	-2	0	0	(1,1,2)	0	0
	(1,1)	-2	-2	0	2	2	0	0	(1,1,2)	0	0
	(1,1)	-2	2	0	2	-2	0	0	(1,1,2)	0	0

C D-Flat Basis for Anomalous SU421 Model

N	Q_A	ϕ						S									
		1	2	3	4	5	6	1	2	3	4	5	6	7	8	9	10
D_1	-8	2	0	0	0	0	0	0	0	0	0	-1	-1	0	-2	-1	1
D_2	-8	0	2	0	0	0	0	0	0	0	0	1	1	0	2	1	-1
D_3	-8	0	0	2	0	0	-2	0	0	0	0	-1	-1	0	-2	-1	1
D_4	-8	0	0	0	2	0	2	0	0	0	0	1	1	0	2	1	-1
D_5	0	0	0	0	0	1	-1	0	0	0	0	-1	-1	0	-2	-1	1
D_6	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	-1	0	1
D_7	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	-1	0	1
D_8	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1

The 8 elements of a basis set of VEVs for forming D -flat directions for the anomalous SU421 model. First column entries give basis element designations. The second column specifies the anomalous charge of each element. The remaining columns specify the ratio of the norms of the VEVs of the scalar fields. All 16 fields are part of vector-like pairs of scalars and a negative norm indicates the vector partner field acquires the VEVs. For each of the D_i in the table there are corresponding \bar{D}_i with sets of norms of VEVs of opposite signs.

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